Fast Coalgebraic Bisimilarity Minimization

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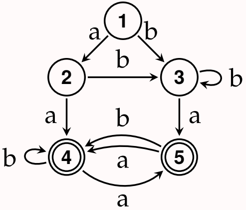
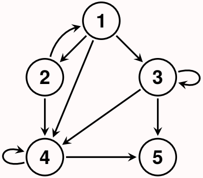
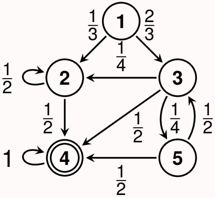
The Automata Zoo

Deterministic finite automata, tree automata, (labeled) transition systems, weighted and probabilistic automata, Markov decision processes, ...







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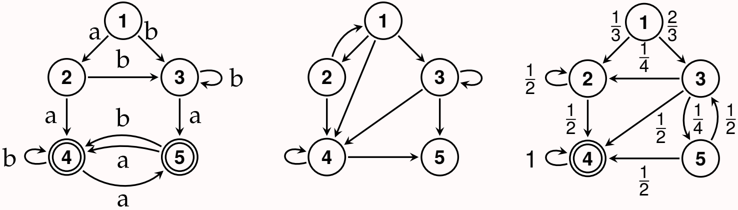

F-coalgebra

a unifying theory of automata and strong bisimilarity

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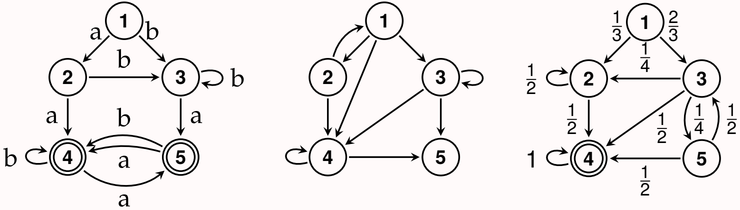

F-coalgebra

a unifying theory of automata and strong bisimilarity

This Work

a **fast** and **general** algorithm for minimizing automata

What's an F-coalgebra?

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Definition

Let $F : \mathcal{C} \longrightarrow \mathcal{C}$ be an endofunctor on a category $\mathcal{C}$. An F**-coalgebra** is an object A of $\mathcal{C}$ together with a morphism $\alpha : A \longrightarrow FA$ of $\mathcal{C}$, usually written as (A, α). An F-coalgebra homomorphism from (A, α) to another F-coalgebra (B, β) is a morphism $f : A \longrightarrow B$ in $\mathcal{C}$ such that $Ff \circ \alpha = \beta \circ f$. Thus the F-coalgebras for a given functor F constitute a category.

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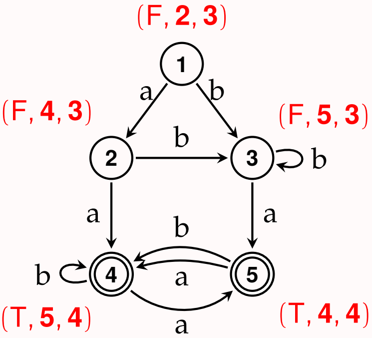
Finite coalgebras unify automata



DFA

$C \to \{F, T\} \times C \times C$

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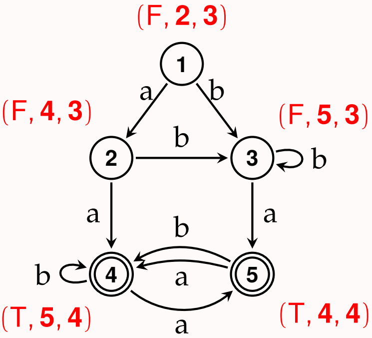

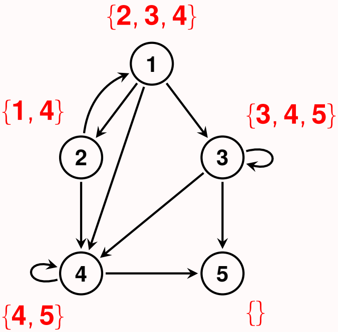

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Transition system

$C \to \mathcal{P}(C)$

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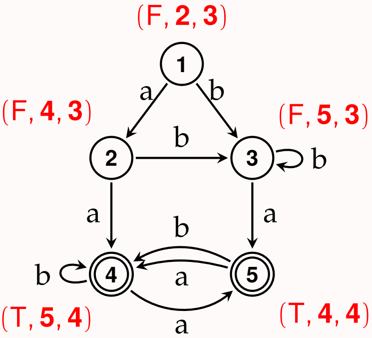

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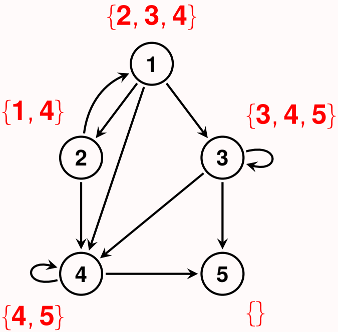


Transition system

$C \to \mathcal{P}(C)$

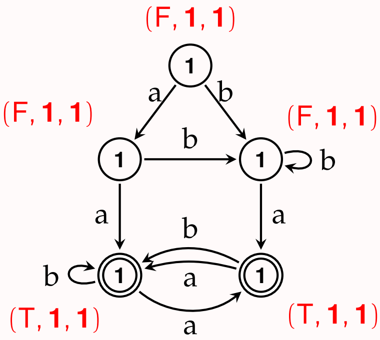
Labeled transition system

$C \to \mathcal{P}(A \times C)$

Markov Decision Process

$C \to \mathcal{P}(\mathcal{D}(C))$

Minimizing a DFA



- Set all the state numbers to 1.
- Pick equivalence class
  - Compute missing signatures.
  - Assign new state numbers & Remove signatures from predecessors of changed states.
- Iterate until all states have a signature.

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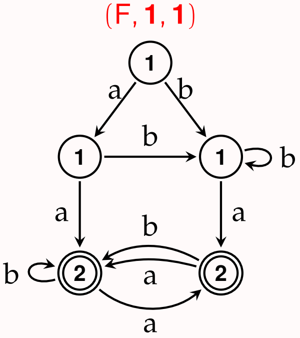

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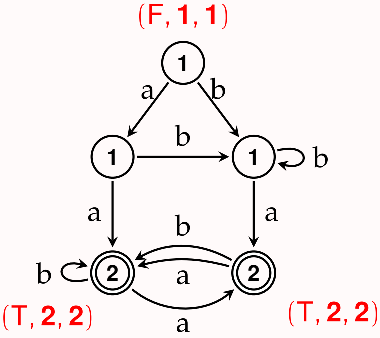

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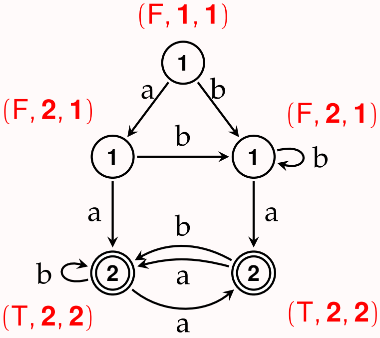

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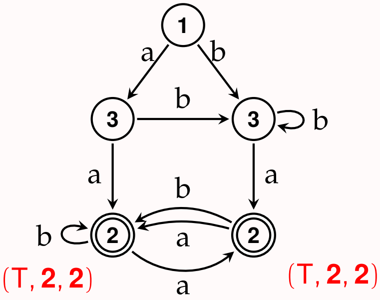

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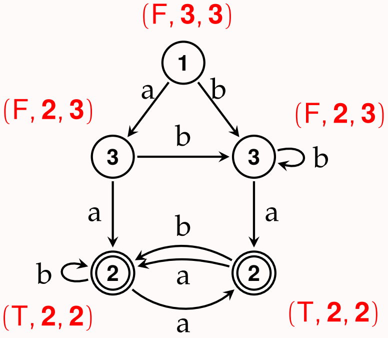

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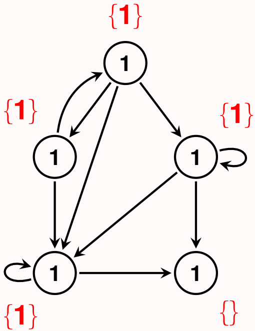

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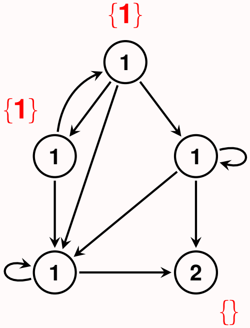

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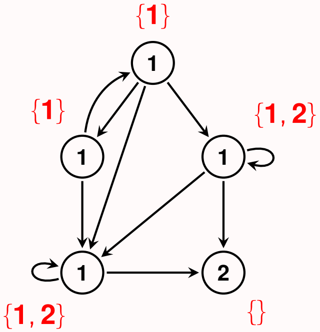

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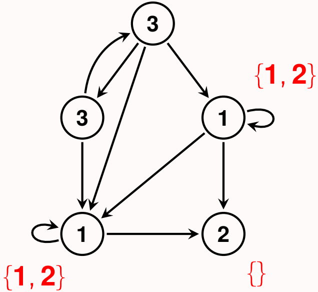

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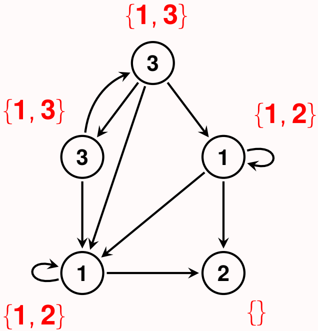

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- What about the complexity of bookkeeping?
  - See paper for n-way partition refinement data structure

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mCRL2 Specialized C++ algorithm suite
for labeled transition systems

$$C \to \mathcal{P}(A \times C)$$

Several LTS minimization algorithms
from literature ('90, '03, '17, '19)

benchmark		**time (s)**			**memory (MB)**	
type	**n**	*CoPaR*	*DCPR*	*Boa*	*DCPR*	*Boa*
fms	**1639440**	232	84	1.12	514×32	196
	4459455	–	406	4.47	1690×32	582
wlan	**607727**	105	855	0.28	147×32	42
	1632799	–	2960	0.79	379×32	93
wta(W)	**152107**	566	79	0.74	642×32	83
	944250	–	675	11.96	6786×32	1228
wta(Z)	**156913**	438	82	0.48	677×32	92
	1007990	–	645	16.75	5644×32	1325
wta(2)	**154863**	449	160	0.81	621×32	79
	1300000	–	1377	23.35	7092×32	1647

What is the cost of generality?

benchmark		time (s)		memory (MB)	
type	**n**	*mCRL2*	*Boa*	*mCRL2*	*Boa*
cwi	**2416632**	13.9	1.4	1780	249
	7838608	214.2	15.8	5777	814
	33949609	282.2	31.5	16615	2776
vasy	**6020550**	33.8	3.1	2124	520
	11026932	51.6	6.1	2768	619
	12323703	56.9	7.0	3103	734

For *mCRL2*, we pick its best algorithm and its self-reported time.
For *Boa*, we report wall-clock time.