A Simple Concurrent Lambda Calculus For Encoding Session types

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Usual message passing:

- Stream of messages of fixed type
- e.g., Go, Rust

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Session types:

- Flexible message passing protocols
- Type of message can depend on the state of the protocol

Two flavours

π calculus: "everything is a channel"

- Elegant minimalist session types
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This work: ƛ calculus = λ calculus + barriers

- Minimal concurrent extension of linear λ calculus
- Local encoding of session types as function types
- Simpler meta theory
- Minimal basis for extensions? (*e.g.*, priorities, sharing)

Session types in GV (Gay, Vasconcelos, Wadler, . . .)

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Linear λ calculus: $\tau ::= \mathbf{0} \mid \mathbf{1} \mid \tau + \tau \mid \tau \times \tau \mid \tau \multimap \tau$

Session types: $s ::= {!}\tau.s \mid {?}\tau.s \mid s \oplus s \mid s \,\&\, s \mid \mathsf{End}$

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\textbf{send} &: ({!}\tau.s) \times \tau \multimap s \\
\textbf{receive} &: ({?}\tau.s) \multimap (s \times \tau) \\
\textbf{tell}_{\mathbf{L}} &: (s_1 \oplus s_2) \multimap s_1 \\
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“Session types”: $\lambda\!\!\!\!-$

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$$\boxed{\alpha \multimap \beta}$$

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Single use!

$$\textbf{let}\ x' = \textbf{fork}(\lambda x.\ x\ 1)\ \textbf{in}\ 1 + x'\ ()$$

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Operational semantics

$$\rho \in \mathsf{N} \xrightharpoonup{\text{fin}} \mathsf{Thread}(e) \mid \mathsf{Barrier}$$

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$$\{n \mapsto \mathsf{Thread}(K[\,e_1\,])\} \rightsquigarrow \{n \mapsto \mathsf{Thread}(K[\,e_2\,])\} \quad \text{if } e_1 \rightsquigarrow_{\text{pure}} e_2$$

$$\{n \mapsto \mathsf{Thread}(K[\,\mathbf{fork}(v)\,])\} \rightsquigarrow \left\{\begin{array}{l} n \mapsto \mathsf{Thread}(K[\ \langle k\rangle\ ]) \\ k \mapsto \mathsf{Barrier} \\ m \mapsto \mathsf{Thread}(v\ \ \langle k\rangle) \end{array}\right\} \qquad (\textbf{fork})$$

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$$\{n \mapsto \mathsf{Thread}(())\} \rightsquigarrow \{\} \qquad (\text{exit})$$

$$\rho_1 \uplus \rho' \rightsquigarrow \rho_2 \uplus \rho' \quad \text{if } \rho_1 \rightsquigarrow \rho_2 \qquad (\text{frame})$$

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&\textbf{let } x' = \textbf{fork}(\lambda x.\ \textbf{let } (y, n) = x\ ()\\
&\qquad\qquad\qquad\qquad\quad \textbf{in } y\ (n \bmod 2 \equiv 0))\\
&\textbf{let } y' = \textbf{fork}(\lambda y.\ x'\ (y, 3))\\
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$\boxed{T_0}$

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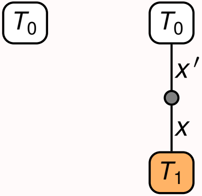

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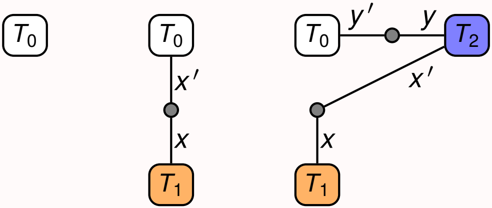

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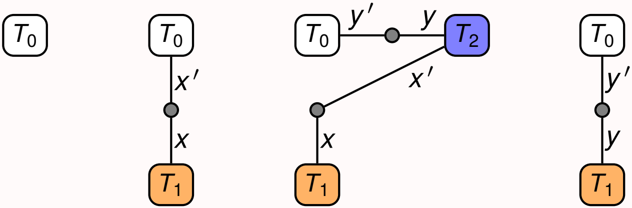

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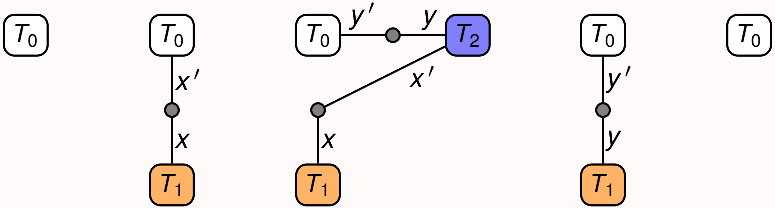

Channel operations as macros

$$\begin{aligned}
\mathbf{fork}_{\text{chan}}(f) &\triangleq \mathbf{fork}(f) \\
\mathbf{send}(c, x) &\triangleq \mathbf{fork}(\lambda c'.\ c\ (c', x)) \\
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Session types as linear function types

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[\![\mathrm{End}]\!] &\triangleq \mathbf{1} \multimap \mathbf{1} \\
[\![\mathbf{!}\tau.s]\!] &\triangleq [\![\overline{s}]\!] \times \tau \multimap \mathbf{1} \\
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\mathbf{send} &: [\![\mathbf{!}\tau.s]\!] \times \tau \multimap [\![s]\!] && \triangleq \lambda(c, x).\, \mathbf{fork}(\lambda c'.\, c\ (c', x)) \\
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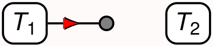
Theorem. If GV program is well-typed, then macro expanded $\lambda\!\!\!^{-}$ program is well-typed

Theorem. Macro expanded $\lambda\!\!\!^{-}$ program simulates GV program

Deadlock freedom: linearity

$\textbf{let } x' = \textbf{fork}(\lambda x.\ ())\ \textbf{in } x'\ 0$ Deadlock!



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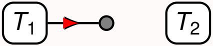
$$\textbf{let } x' = \textbf{fork}(\lambda x.\, ())\ \textbf{in } x'\ 0$$ Deadlock!



Ruled out by linear typing

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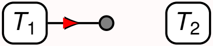
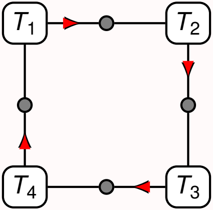
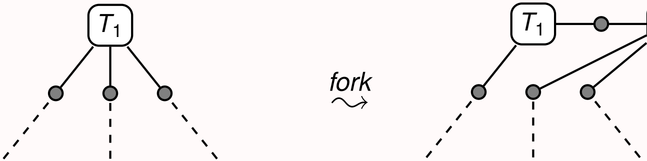


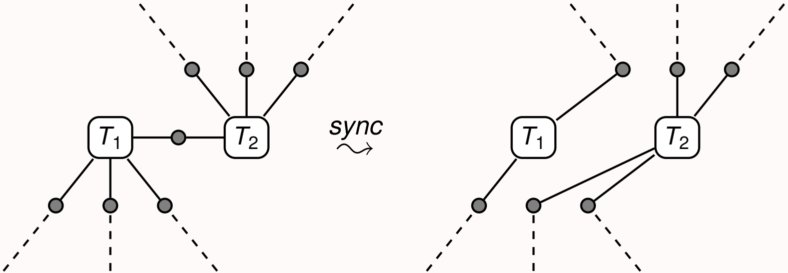
Ruled out by linear typing

But what about cycles?



Deadlock freedom: acyclicity





Mechanized proofs in Coq

Meta theory of ƛ + recursive types + non-linear types

- Global progress:
  $(e : 1) \wedge \{0 \mapsto e\} \leadsto \rho \implies \rho \text{ can step} \vee \rho = \{\}$
- Partial deadlock freedom
- Memory leak freedom
- Size $\approx \frac{1}{2}$ earlier GV mechanization

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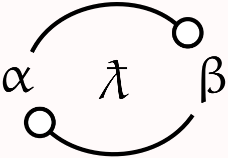
Session types in $\lambda\!\!\!\!^-$

- Compiler from GV to $\lambda\!\!\!\!^-$
- Proof that output $\lambda\!\!\!\!^-$ program is well-typed
- Proof that output $\lambda\!\!\!\!^-$ program simulates GV program

$$\underbrace{\textbf{fork} : ((\alpha \multimap \beta) \multimap \mathbf{1}) \multimap (\beta \multimap \alpha)}_{\text{Session types distilled}}$$



Questions?

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