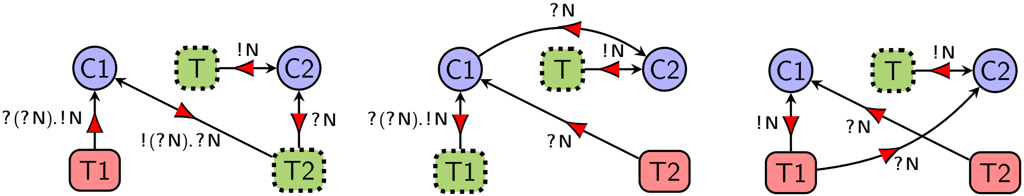
Connectivity Graphs: A Method for Proving Deadlock Freedom Based on Separation Logic

Jules Jacobs[1] Robbert Krebbers[1] Stephanie Balzer[2]



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POPL'22/plunch

Session types

Message passing concurrency with first-class channels (Honda [1993])

$$c : !\mathit{Nat}.?\mathit{Bool}.!(?\mathit{String}.!\mathit{Nat}.\mathsf{End}).\mathsf{End}$$

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$$\Updownarrow \text{dual}$$

$$c' : ?Nat.\,!Bool.\,?(?String.\,!Nat.\,\mathrm{End}).\,\mathrm{End}$$

GV: functional programming with session types
(Gay and Vasconcelos [2010], Wadler [2012])

$$\mathrm{fork} : (s \xrightarrow{lin} 1) \to \overline{s} \qquad \mathrm{send} : (!t.\,s) \times t \xrightarrow{lin} s$$

$$\mathrm{close} : \mathrm{End} \xrightarrow{lin} 1 \qquad \mathrm{receive} : ?t.\,s \xrightarrow{lin} s \times t$$

$$\text{let } c = \mathrm{fork}(\lambda c'.\ ...\mathrm{receive}(c')...) \text{ in } \mathrm{send}(c, 23)...$$

What makes session types interesting

Linear session types: cannot copy or delete a channel reference before you are done

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- Required for **type safety** (mechanized by Castro-Perez et al. [2020], Ciccone and Padovani [2020], Goto et al. [2016], Hinrichsen et al. [2021], Rouvoet et al. [2020], Thiemann [2019], ...)

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- But also guarantees **deadlock freedom**, **global progress** (well-known property – Caires & Pfenning, Wadler, Carbone – but not yet mechanized for first-class channels, i.e. dynamically allocated and higher order)

Why session types give deadlock freedom

Two owners per channel

- Duality of channel types → no simple deadlocks
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Difficult to reason about typing & graph structure simultaneously

Contribution: connectivity graph proof method

This work: connectivity graphs

- Method for factoring out graph reasoning from reasoning about typing
- Mechanized in the Coq proof assistant
- Applied to prove deadlock freedom for feature-rich session-typed language
- Abstract representation of run-time configuration

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Run-time configuration ρ

Threads: $\{T_1 \mapsto e_1, ..., T_6 \mapsto e_6\}$
Channels: $\{C_1 \mapsto \mathrm{buf}_1, ..., C_5 \mapsto \mathrm{buf}_5\}$

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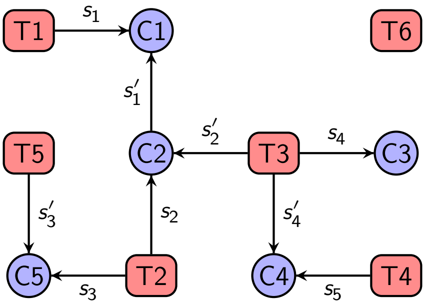
Connectivity graph G



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$$G \vDash \rho$$
$$wf(\rho) := \exists G. G \vDash \rho$$

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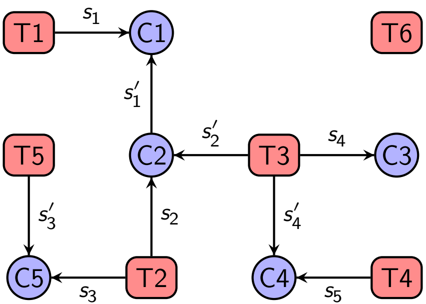

Connectivity graph proof based on progress and preservation

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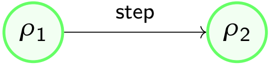

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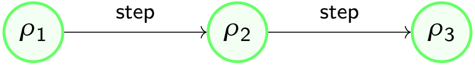

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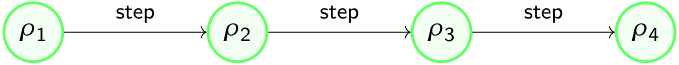

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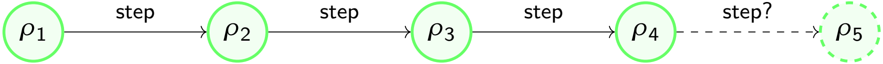

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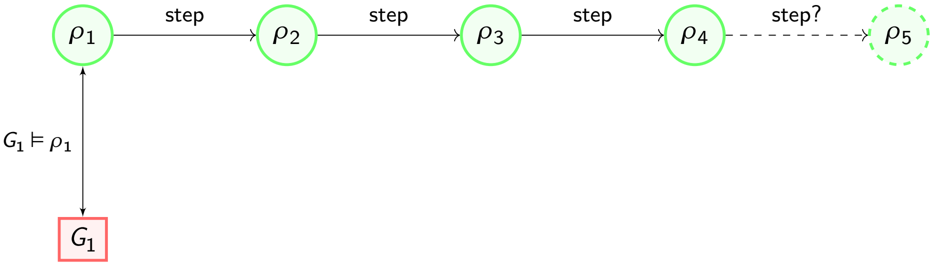

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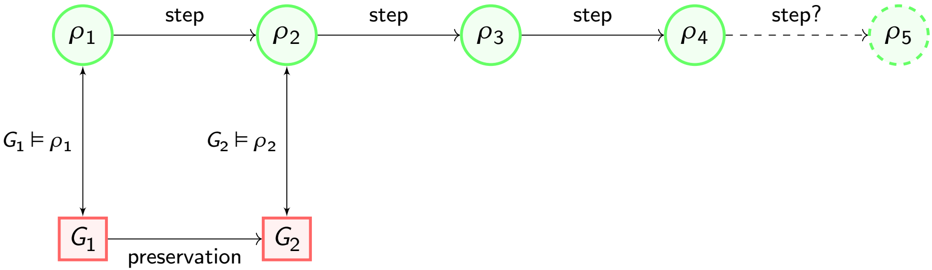

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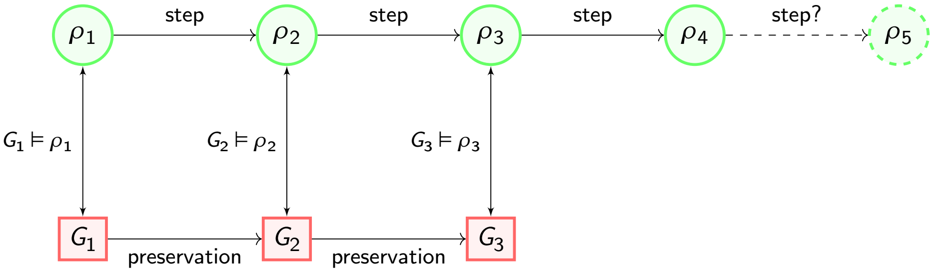

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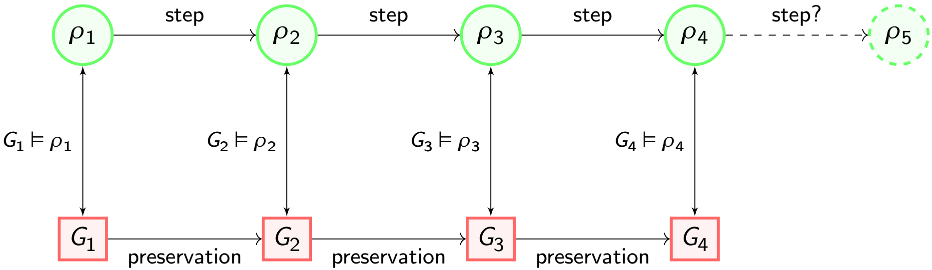

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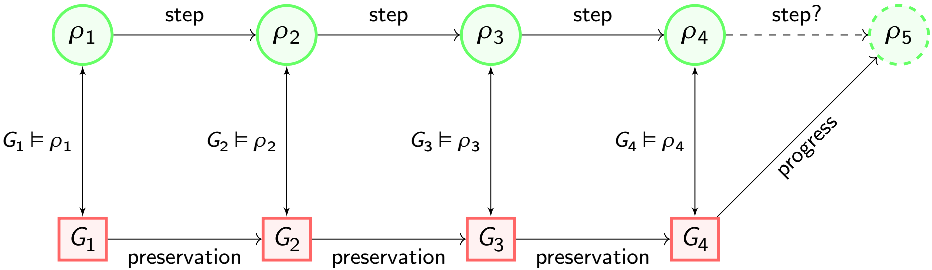

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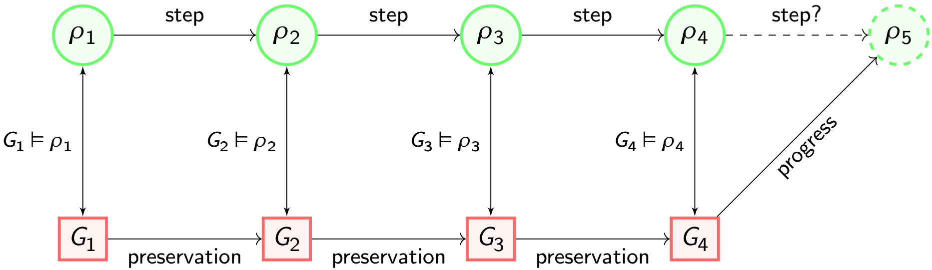

Connectivity graph framework:

- $Cgraph(V, L)$ data type for acyclic labeled graphs
- Generic construction for $wf(\rho) := \overline{wf}(P_\rho)$
  - Parameterized by local separation logic predicate $P_\rho(v)$ for each vertex $v \in G$
- Preservation: graph transformations in separation logic
- Progress: waiting induction principle for $Cgraph(V, L)$

All generic over vertices V and labels L

Linear heap typing in separation logic: (cf. Rouvoet [2020]'s definitional interpreters)

$$\frac{\Sigma_1 \vdash e_1 : \tau_1 \qquad \Sigma_2 \vdash e_2 : \tau_2 \qquad \Sigma_1 \cap \Sigma_2 = \emptyset}{\Sigma_1 \cup \Sigma_2 \vdash (e_1, e_2) : \tau_1 \times \tau_2}$$

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$$\frac{\Sigma = \{\mathsf{Chan}(a) \mapsto (t, s)\}}{\Sigma \vdash \#a_t : s} \quad \Rightarrow \quad \frac{\mathsf{own}(\mathsf{Chan}(a) \mapsto (t, s))}{\#a_t : s}*$$

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Lemmas in separation logic:

$$(\Sigma \vdash K[e] : B) \iff \exists A, \Sigma_1, \Sigma_2.\ (\Sigma_1 \cap \Sigma_2 = \emptyset) \land (\Sigma = \Sigma_1 \cup \Sigma_2) \land (\Sigma_1 \vdash e : A) \land$$
$$\forall e', \Sigma_3.\ (\Sigma_2 \cap \Sigma_3 = \emptyset) \land (\Sigma_2 \vdash e' : A) \rightarrow (\Sigma_2 \cup \Sigma_3 \vdash K[e'] : B)$$

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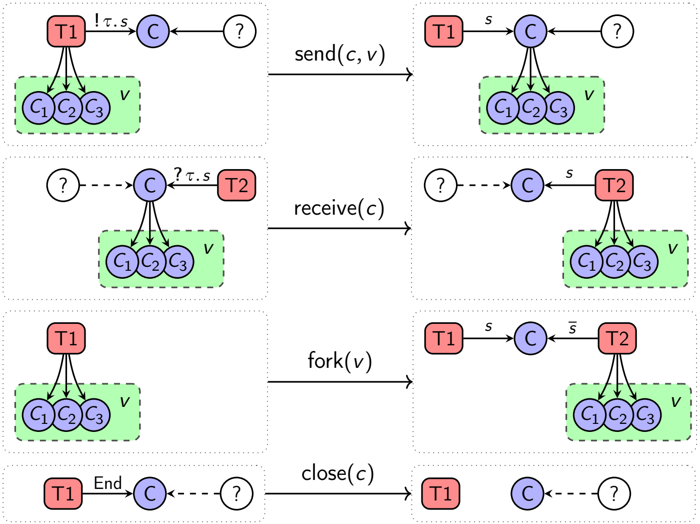
$$\forall e', \Sigma_3.\ (\Sigma_2 \cap \Sigma_3 = \emptyset) \land (\Sigma_2 \vdash e' : A) \to (\Sigma_2 \cup \Sigma_3 \vdash K[e'] : B)$$

$$\Rightarrow$$

$$(K[e] : B) \;\; \dashv\vdash \;\; \exists A.\ (e : A) * \forall e'.\ (e' : A) \mathrel{-\!\!*} (K[e'] : B)$$

We use the Iris proof mode to reason in separation logic (Krebbers et al. [2017])

Preservation via local graph transformations



Preserves:

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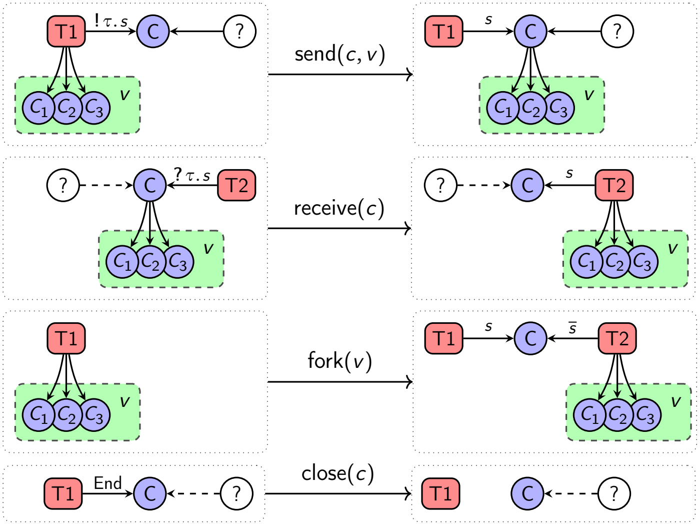

Preserves:

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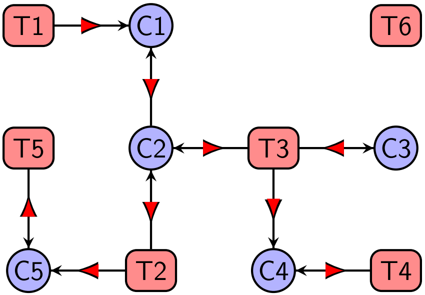
In separation logic: if

$$P_\rho(T_1) * (\mathrm{own}(C \mapsto \ell) \twoheadrightarrow P_\rho(C))$$
$$\vdash$$
$$(\mathrm{own}(C \mapsto \ell') \twoheadrightarrow P_{\rho'}(T_1)) * P_{\rho'}(C)$$

then: $\overline{wf}(P_\rho) \to \overline{wf}(P_{\rho'})$
Explained in our paper!

Progress via waiting induction

Connectivity graph with *waiting dependencies* (▶)
derived from run-time configuration



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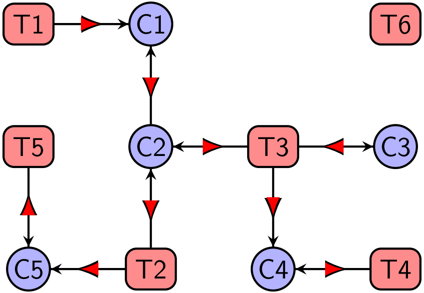

Lemma (Waiting induction)

Let $R(v, w)$ be any relation on the vertices. To prove $P(v)$, we may assume $P(w)$ for all w such that $v \longrightarrow w$ and $R(v, w)$, or $w \longrightarrow v$ and $\neg R(w, v)$

Our language

Functional language + session-typed channels (extension of Wadler [2012]'s GV)

Unrestricted and linear types

- Unrestricted: numbers, sums, products, unrestricted function type ($\rightarrow$)
- Linear: channels, sums, products, linear function type ($\multimap$)

General recursive types:

- Recursive session types, including through the message (example: $\mu X.\ !X.End$)
- Algebraic data types using recursion + sums + products
- Recursive types mechanized using coinduction (Gay et al. [2020])

Stronger deadlock and leak freedom result

Global progress is the standard notion that people use

Our POPL reviewers: Can your method prove something stronger?

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Partial deadlock: a set S of threads and channels such that:

1. All threads in S are blocked on a channel in S
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Lemma. All threads and channels are reachable $\iff$ no partial deadlock
Lemma. Any thread or channel is reachable $\implies$ global progress
Theorem. For well-typed initial programs, no partial deadlock occurs

Mechanization

Mechanization in Coq:

- Generic $\mathit{Cgraph}(V, L)$ library: 4999 LOC
- GV language definition: 451 LOC
- Language specific deadlock and leak freedom proof: 1688 LOC

`https://github.com/julesjacobs/cgraphs`

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Initial direct attempt: **proofs goals got too complex.**
Graph reasoning intertwined with language specifics.
Encapsulating the graph reasoning made it manageable.

Other deadlock freedom proofs

μ**GV**

- Linear lambda calculus + fork with single-shot atomic exchange
- $\mathit{fork} : ((a \xrightarrow{\mathit{lin}} b) \xrightarrow{\mathit{lin}} 1) \rightarrow (b \xrightarrow{\mathit{lin}} a)$
- Global progress & deadlock freedom in Coq (1478 LOC)

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Multiparty session types

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  1. Single-session deadlock freedom
  2. Lock-order based deadlock freedom

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Multiparty session types

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- Multiparty literature has focused on:
  1. Single-session deadlock freedom
  2. Lock-order based deadlock freedom
- **MPGV**:
  1. Deadlock freedom for *all* well-typed programs
  2. MPGV multiparty session types > binary session types
  3. Global progress & deadlock freedom in Coq (10400 LOC)

Questions?

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